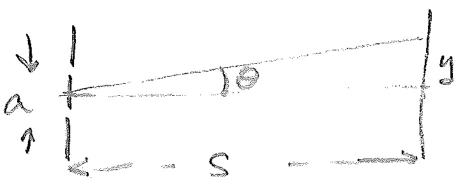
P302 Photonics Fall 2010
Solutions to HW # 11

(12.2)



$a =$ distance between slits
$b =$ width of slits

According to pg 294, if $E_{0_1} = E_{0_2}$ (have same amplitude)

$$E_1 + E_2 = E_0(x,t)\cos(\bar{k}x - \bar{\omega}t)$$

with $E_0(x,t) = 2E_{0_1}\cos(k_m x - \omega_m t)$

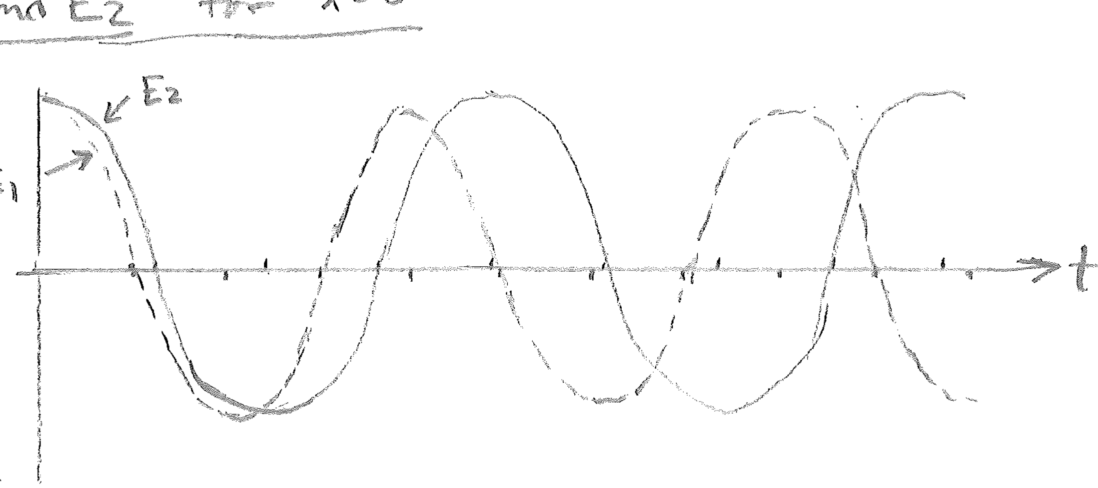
$\left\{\begin{array}{l} \bar{k} = \frac{1}{2}(k_1 + k_2) \\ \bar{\omega} = \frac{1}{2}(\omega_1 + \omega_2) \end{array}\right\}$ are the averages wave number & frequency

$\left\{\begin{array}{l} k_m = \frac{1}{2}(k_2 - k_1) \\ \omega_m = \frac{1}{2}(\omega_2 - \omega_1) \end{array}\right\}$ are the modulated (beat) wave number + frequency

Since $\lambda_1 = 0.8\lambda_2 \Rightarrow \omega_1 = \frac{\omega_2}{0.8} = 1.25\omega_2$

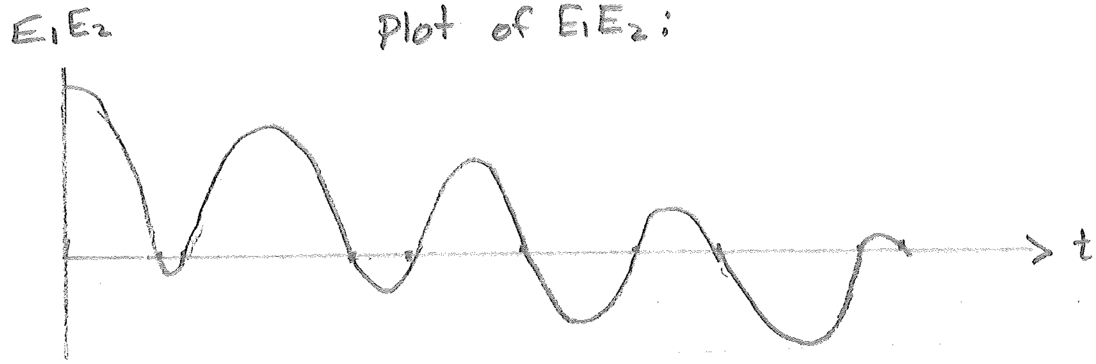
and $T_1 = 0.8 T_2$

Plot E_1 and E_2 ~~for~~ $x = 0$



(12.2)

Plot of E_1E_2:



We can write E_1E_2 in another form:

let $E_1 = E_{0_1}\cos(k_1x - w_1t) \equiv E_{0_1}\cos\alpha_1$

So $E_2 = E_{0_2}\cos(k_2x - w_2t) \equiv E_{0_2}\cos\alpha_2$

For $E_{0_1} = E_{0_2}$

$E = E_1 + E_2 = E_{0_1}(\cos\alpha_1 + \cos\alpha_2)$

So $E_1E_2 = 2E_{0_1}^2\cos\alpha_1\cos\alpha_2$

But $\cos\alpha_1\cos\alpha_2 = \frac{1}{2}[\cos(\alpha_1+\alpha_2) + \cos(\alpha_1-\alpha_2)]$

So $\langle E_1E_2\rangle_T = E_{0_1}^2\langle[\cos(\alpha_1+\alpha_2) + \cos(\alpha_1-\alpha_2)]\rangle_T$

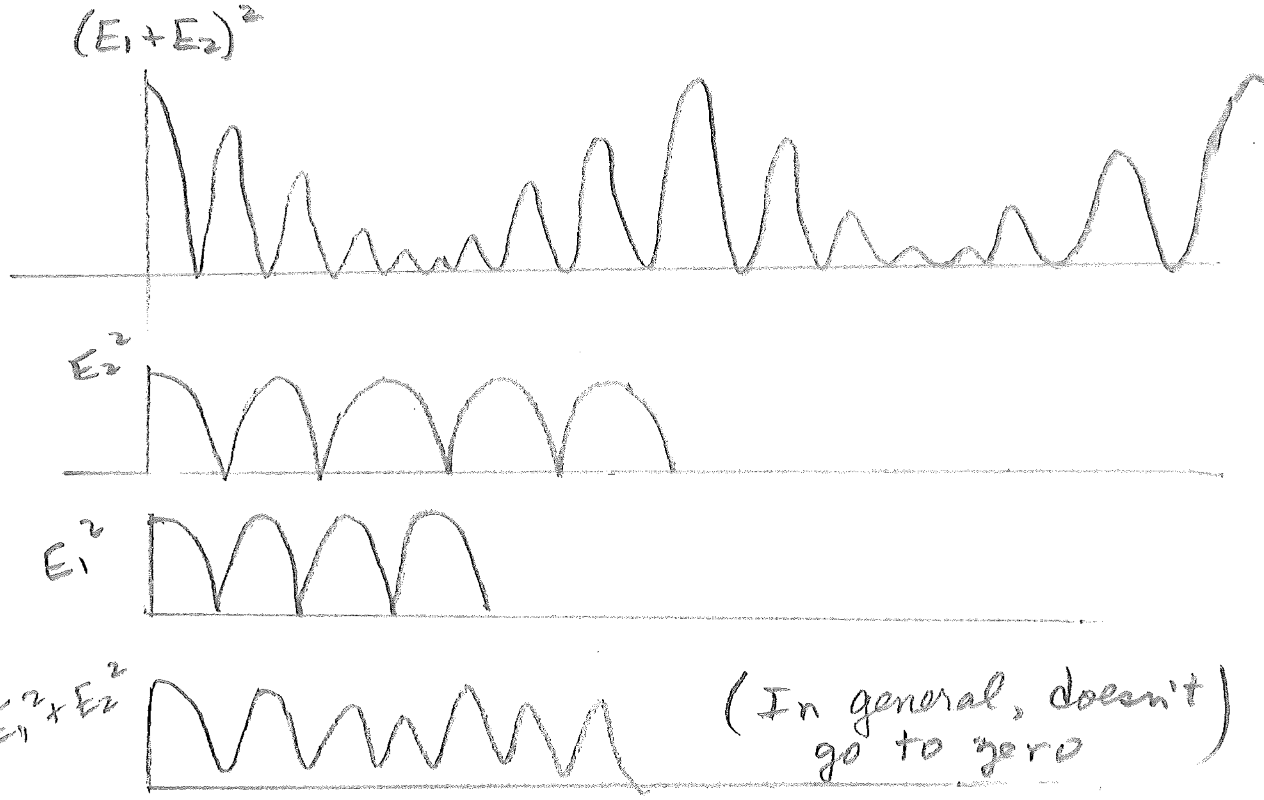
Remember $\alpha_1 \pm \alpha_2 = (k_1 \pm k_2)x - (w_1 \pm w_2)t$

and $\langle\cos(kx-wt)\rangle_T = 0$

$\boxed{\text{Thus } \langle E_1E_2\rangle_T = 0 \text{ if you average over a long time.}}$

12.2

From pg 294, we can plot $(E_1+E_2)^2$ (using a different scale than above)



Now approximate $\langle (E_1+E_2)^2\rangle_T = \langle E_1^2\rangle_T + \langle E_2^2\rangle_T + 2\langle E_1E_2\rangle_T$

Since $\langle E_1E_2\rangle_T \cong 0$, we expect

$$\langle (E_1+E_2)^2\rangle_T \cong \langle E_1^2\rangle_T + \langle E_2^2\rangle_T$$

(12.5) $\mathcal{V} = \dfrac{I_{max}/I - I_{min}/I}{I_{max}/I + I_{min}/I}$. Using eqs 12.8 + 12.9:

$$\mathcal{V} = \left| \operatorname{sinc}\left(\frac{a\pi w}{s\lambda}\right) \right|$$

The max occur when $\dfrac{a\pi w_n}{s\lambda} = \left(\dfrac{2n-1}{2}\right)\pi$

so $\Delta w = w_{n+1} - w_n$

$$\Delta w = \frac{s\lambda}{a} \overset{set}{=} 1\ mm \quad \left(\begin{matrix}\text{max-to-max} \\ \text{seperation}\end{matrix}\right)$$

Since $w = 0.5mm$

$$\mathcal{V} = \left| \operatorname{sinc}\left(\frac{\pi(0.5mm)}{1mm}\right) \right| = \left| \operatorname{sinc} \frac{\pi}{2} \right|$$

or $\boxed{\mathcal{V} = \frac{2}{\pi}}$

(12.9) When the narrow slit source (width b) is replaced with a circular aperture source, the visibility becomes proportional to the first order Bessel function (see pages 565-566), ie

$$\mathcal{V} \propto J_1\left(\frac{a\pi w}{s\lambda}\right)$$

The first zero of J_1 occurs at 3.83, so $\mathcal{V} = 0$ when

$$\frac{a\pi w}{s\lambda} = 3.83 \quad . \text{ But } \frac{w}{s} = \frac{(2b)}{l}$$

where b = radius of circular aperture source, ie 2b = diameter = 0.1 mm

So $$\frac{a\pi(2b)}{l\lambda} = 3.83$$

or $$a = \frac{3.83\, l\lambda}{\pi(2b)} = \frac{(3.83)(1\text{m})(589.3\text{nm})}{\pi(0.1\text{mm})}$$

$$\underline{a = 7.18\text{ mm}}$$